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# More Modular Invariant Anomalous $U(1)$ Breaking\*

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## Abstract

We consider the case of several scalar fields, charged under a number of  $U(1)$  factors, acquiring vacuum expectation values due to an anomalous  $U(1)$ . We demonstrate how to make redefinitions at the superfield level in order to account for tree-level exchange of vector supermultiplets in the effective supergravity theory of the light fields in the supersymmetric vacuum phase. Our approach builds upon previous results that we obtained in a more elementary case. We find that the modular weights of light fields are typically shifted from their original values, allowing an interpretation in terms of the preservation of modular invariance in the effective theory. We address various subtleties in defining unitary gauge that are associated with the noncanonical Kähler potential of modular invariant supergravity, the vacuum degeneracy, and the role of the dilaton field. We discuss the effective superpotential for the light fields and note how proton decay operators may be obtained when the heavy fields are integrated out of the theory at the tree-level. We also address how our formalism may be extended to describe the generalized Green-Schwarz mechanism for multiple anomalous  $U(1)$ 's that occur in four-dimensional Type I and Type IIB string constructions.

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# 1 Introduction

In our previous article [1] we studied the effective supergravity theory obtained in the presence of an anomalous  $U(1)$ , that for the remainder of this article we will denote  $U(1)_X$ . In the simple case that we investigated, a single scalar field charged under  $U(1)_X$  acquired a nonvanishing vacuum expectation value (*vev*). The associated chiral multiplet was “eaten” by the  $U(1)_X$  vector multiplet to form a massive vector multiplet. We eliminated tree-level exchange of this massive vector multiplet by redefinitions that eliminated linear couplings between the heavy and light fields. We demonstrated that this redefinition can be made at the superfield level, while maintaining manifest modular invariance and the (modified) linearity conditions,

$$(\bar{\mathcal{D}}^2 - 8R)L = - \sum_a (\mathcal{W}\mathcal{W})_a, \quad (\mathcal{D}^2 - 8\bar{R})L = - \sum_a (\overline{\mathcal{W}\mathcal{W}})_a, \quad (1.1)$$

for the linear superfield  $L$ , whose lowest component is the real scalar associated with the dilaton. A comparison with redefinitions at the component field level provided assurances that the superfield approach was reliable.

Our motivations stemmed from the prevalence of a  $U(1)_X$  factor in the string scale gauge group of semi-realistic string compactifications; for example, in a recent study [2] of a certain class of standard-like heterotic  $Z_3$  orbifold models, it was found that 168 of 175 models had an anomalous  $U(1)_X$ . Clearly the simple case considered in our previous article does not address the complications that arise in the semi-realistic models that we seek to understand, since the scalars that get *vev*'s due to the  $U(1)_X$  are typically charged under several  $U(1)$  factors and multiple scalars must generally get *vev*'s in order for the *D-terms* of the several  $U(1)$ 's to (approximately) vanish. The purpose of this paper is to examine the effective supergravity theory when these two generalizations are made, for the case of the supersymmetric vacuum phase. As has been noted previously, the supersymmetric vacuum is approximately the stable vacuum in the case where dynamical supersymmetry breaking *via* gaugino condensation occurs in an effective supergravity context [3]. Thus the scenario studied here represents a *bona fide* starting point for the effective supergravity theory of semi-realistic models with a  $U(1)_X$ .

We start with the effective theory at the string scale defined as in [1]:

$$\mathcal{L} = \int d^4\theta \tilde{L} + \mathcal{L}_Q + \mathcal{L}_{th}, \quad (1.2)$$

where  $\tilde{L}$  is the real superfield functional

$$\tilde{L} = E [-3 + 2Ls(L) + L(bG - \delta_X V_X)] = E [-3 + 2LS], \quad (1.3)$$

and where the Kähler potential given by

$$\begin{aligned} K &= k(L) + G + \sum_A e^{G^A + 2 \sum_a q_A^a V^a} |\Phi^A|^2, \quad G = \sum_I g^I, \quad G^A = \sum_I q_I^A g^I, \\ g^I &= -\ln(T^I + \bar{T}^I), \quad k(L) = \ln L + g(L). \end{aligned} \quad (1.4)$$

In the dual chiral formulation  $s(L) \rightarrow \text{Res}$ ;  $\langle s(L) \rangle = g^{-2}$  at the string scale. Canonical normalization of the Einstein term requires:

$$k'(L) = -2Ls'(L). \quad (1.5)$$

For reasons explained in [1], we are not using  $U(1)_K$  superspace for the Abelian gauge groups that are broken at the string scale by the anomalous  $U(1)_X$ .

Since the underlying theory is anomaly free, it is known [4] that the apparent anomaly is canceled by a four-dimensional version of the Green-Schwarz (GS) mechanism [5]. This leads to a Fayet-Illiopoulos (FI) term in the effective supergravity Lagrangian. Ignoring nonperturbative corrections to the dilaton Kähler potential,

$$D_X = \sum_A \frac{\partial K}{\partial \phi^A} q_A^X \phi^A + \xi, \quad \xi = \frac{g_s^2 \text{Tr } T_X}{192\pi^2} m_P^2, \quad (1.6)$$

where  $K$  is the Kähler potential,  $q_A^X$  is the  $U(1)_X$  charge of the scalar matter field  $\phi^A$ ,  $\xi$  is the FI term,  $T_X$  is the charge generator of  $U(1)_X$ ,  $g_s$  is the unified (string scale) gauge coupling, and  $m_P = 1/\sqrt{8\pi G} = 2.44 \times 10^{18}$  GeV is the reduced Planck mass. In the remainder we work in units where  $m_P = 1$ .

Up to perturbative loop effects, the chiral dilaton formulation has  $g_s^2 = 1/\text{Re}\langle s \rangle$ , where  $s = S|$  is the lowest component of the chiral dilaton superfield  $S$ . However, once higher order and nonperturbative corrections are taken into account the chiral dilaton formulation becomes inconvenient. The dual linear multiplet formulation—that relates a (modified) linear superfield  $L$  to  $\{S, \bar{S}\}$  through a duality transformation—provides a more convenient arrangement of superfield degrees of freedom due to the neutrality of  $L$  with respect to target-space duality transformations (hereafter called *modular transformations*). In the limit of vanishing nonperturbative corrections to the dilaton Kähler potential,  $g_s^2 = 2\langle \ell \rangle$ , where  $\ell = L|$ . Throughout this article we use the linear multiplet formulation [6, 7]. Except where noted above, we use the  $U(1)_K$  superspace formalism [8, 7, 9]. (For a review of the  $U(1)_K$  superspace formalism see [9]; for a review of the linear multiplet formulation see [10].)

In the linear multiplet formulation, including nonperturbative corrections to the dilaton Kähler potential, the FI term becomes

$$\xi(\ell) = \frac{2\ell \text{Tr } T_X}{192\pi^2}. \quad (1.7)$$

Consequently, the background dependence of the FI term in (1.7) arises from  $\langle \ell \rangle = \langle L \rangle$ . The FI term induces nonvanishing *vev*'s for some scalars  $\phi^A$  as the scalar potential drives  $\langle D_X \rangle \rightarrow 0$ , if supersymmetry is unbroken. The nonvanishing *vev*'s in the supersymmetric vacuum phase can be related to the FI term. Then  $\langle L \rangle$  serves as an order parameter for the vacuum and all nontrivial *vev*'s can be written as some fraction of  $\langle L \rangle$ . *Our approach in what follows will be to promote this to a superfield redefinition.* Thus we impose the superfield identity

$$\left( \frac{\partial K}{\partial V_a} + 2L \frac{\partial S}{\partial V_a} \right)_{\Delta_A=0} = \left( \frac{\partial K}{\partial V_a} \right)_{\Delta_A=0} - L \delta_X \delta_{Xa} = 0, \quad (1.8)$$

where  $\Delta_A$  are superfields, to be defined below, that vanish in the supersymmetric vacuum. This assures vanishing of the D-terms at the  $U(1)_a$  symmetry breaking scale while maintaining manifest supersymmetry below that scale. The latter point was demonstrated in detail, at both the superfield and the component field levels, for the toy model studied in [1].

$\mathcal{L}_Q$  is the quantum correction [11, 12, 13] that contains the field theory anomalies canceled by the GS terms:

$$\mathcal{L}_Q = - \int d^4\theta \frac{E}{8R} \sum_a \mathcal{W}_a^\alpha P_\chi B_a \mathcal{W}_\alpha^a + \text{h.c.}, \quad (1.9)$$

$$B_a(L, V_X, g^I) = \sum_I (b - b_a^I) g^I - \delta_X V_X + f_a(L), \quad (1.10)$$

where  $P_\chi$  is the chiral projection operator [14]:  $P_\chi \mathcal{W}^\alpha = \mathcal{W}^\alpha$ , that reduces in the flat space limit to  $(16\Box)^{-1} \bar{\mathcal{D}}^2 \mathcal{D}^2$ , and the  $L$ -dependent piece  $f_a(L)$  is the “2-loop” contribution [11]. The string-loop contribution is [15]

$$\mathcal{L}_{th} = - \int d^4\theta \frac{E}{8R} \sum_{a,I} b_a^I (\mathcal{W}\mathcal{W})_a \ln \eta^2(T^I) + \text{h.c.} \quad (1.11)$$

For each  $\Phi^A$ , the  $U(1)_X$  charge is denoted  $q_A^X$  while  $q_I^A$  are the modular weights. The conventions chosen here imply  $U(1)_X$  gauge invariance under the transformation

$$V_X \rightarrow V'_X = V_X + \frac{1}{2} (\Theta + \bar{\Theta}), \quad \Phi^A \rightarrow \Phi'^A = e^{-q_A^X \Theta} \Phi^A. \quad (1.12)$$

The GS coefficients  $b$  and  $\delta_X$  must be chosen to cancel the quantum field anomalies under modular and  $U(1)_X$  transformations that would be present in the absence of the GS counterterms [4, 16]. It is not hard to check that the correct choices are given by:

$$\delta_X = - \frac{1}{2\pi^2} \sum_A C_{a \neq X}^A q_A^X = - \frac{1}{48\pi^2} \text{Tr } T_X, \quad (1.13)$$

$$8\pi^2 b = 8\pi^2 b_a^I + C_a - \sum_A (1 - 2q_I^A) C_a^A. \quad (1.14)$$

In Section 2 we address complications introduced by the occurrence of several chiral superfields in the theory, some with scalar components getting *vev*'s and others whose scalar components do not get *vev*'s. Unitary gauge and the decomposition of chiral multiplets into light and heavy multiplets is addressed. In Section 3 we discuss the case where the fields getting large *vev*'s due to the  $U(1)_X$  are charged under several  $U(1)$ 's. It is shown how the “eating” of chiral multiplets proceeds in this situation. Further, we look into the reinterpretation of the modified linearity conditions when written in terms of vector superfields with vanishing *vev*'s after the necessary Weyl transformation is made. In Section 4 we investigate the effective superpotential that results when the field redefinitions are made. We demonstrate how new operators are obtained when tree-level exchange is accounted for. As an example of the relevance of such effective operators, we note how proton decay operators may be obtained. In Section 5 we present our conclusions and lay out items that remain to be investigated. In Appendix A we describe the necessary condition to have a canonical Einstein term. In Appendix B we formulate the Weyl transformation necessary to eliminate linear couplings to the heavy fields, maintain the linearity of  $L$  and preserve the canonical normalization of the Einstein term. In Appendix C we extend our formalism to the generalized GS cancellation of several anomalous  $U(1)$ 's that occurs in Type IIB and Type I string theories. Here, several linear multiplets are involved that do not correspond to the dilaton.

## 2 Generalized Field Content

In this section we assume a set of chiral superfields  $\Phi^A$  that are charged only under  $U(1)_X$ . Then the first equality in (1.4) is simply

$$K = k(L) + G + \sum_A K_{(A)}, \quad K_{(A)} = e^{G^A + 2q_A V} |\Phi^A|^2. \quad (2.1)$$

We will encounter subtleties in going to unitary gauge, in part because the sum in (2.1) is weighted by  $\exp G^A$ , which may be different for the various matter fields getting *vev*'s to cancel the FI term. Our emphasis, as in our previous article, will be on keeping modular invariance manifest. We parameterize the vacuum in terms of modular invariant *vev*'s  $\langle K_{(A)} \rangle$ , which by the vanishing of (1.6) in the supersymmetric vacuum we can rewrite in terms of  $\ell$ . We promote this to a superfield redefinition throughout, in order to retain explicit supersymmetry, as well as modular invariance, in the effective theory below the  $V$  mass scale. When there is more than one scalar *vev* contributing to the  $U(1)_X$  gauge symmetry breaking there are further difficulties in this approach, as discussed in Section 2.2, that are

related to the degeneracy of the vacuum, and the role of the linear multiplet as an order parameter.

## 2.1 One Vev

Suppose first that only one field  $\Phi^{A_0} \equiv e^\Theta$ , with  $q_{A_0} = q$ ,  $q_I^{A_0} = q^I$ , gets a *vev*. In this simple case, unitary gauge is obtained as in our previous study; under (1.12) we have

$$V \rightarrow V' = V + \frac{1}{2q} (\Theta + \bar{\Theta}), \quad \Phi^{A_0} \rightarrow \Phi'^{A_0} = e^{-\Theta} \Phi^{A_0} = 1. \quad (2.2)$$

The field  $V'$  describes a massive vector multiplet in the unitary gauge; it has “eaten” the chiral superfield  $\Theta$  and its conjugate. The contribution to the Kähler potential from this field then simplifies to ( $G_q \equiv G^{A_0}$ ):

$$K_{(A_0)} = e^{G_q + 2qV'}. \quad (2.3)$$

For the other chiral superfields we have from (1.12)

$$\Phi^A \rightarrow \Phi'^A = e^{-q_A \Theta / q} \Phi^A \quad (2.4)$$

and the corresponding contributions to the Kähler potential become

$$K_{(A)} = e^{G^A + 2q_A V'} |\Phi'^A|^2 = e^{G^A + 2q_A V'} |\Phi'^A|^2. \quad (2.5)$$

Because  $\langle \Phi^{A_0} \rangle \neq 0$  we see that if we assume<sup>1</sup>  $\langle V \rangle = 0$  we have after the gauge transformation (2.2) that  $\langle V' \rangle \neq 0$ . Moreover, while  $V$  is modular invariant,  $V'$  is not, as is obvious from (2.2) since  $\Theta$  is not modular invariant. It is convenient to work instead with a modular invariant vector superfield with vanishing *vev*; we therefore follow our previous approach and define

$$V' \equiv U + \frac{1}{2q} \left( \ln \frac{\delta_X L}{2q} - G_q \right), \quad \langle U \rangle \equiv 0. \quad (2.6)$$

Going over to this basis, we have

$$K_{(A)} = e^{G'^A + 2q_A U} \left( \frac{\delta_X L}{2q} \right)^{q_A/q} |\Phi'^A|^2, \quad (2.7)$$

$$G'^A = \sum_I q_I'^A g^I, \quad q_I'^A = q_I^A - q_I q_A / q. \quad (2.8)$$

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<sup>1</sup>We can always do this by going to WZ gauge for  $V$ . The same situation is *not* true for  $V'$ .

In particular we have  $q_I'^{A_0} = 0$ , yielding

$$K_{(A_0)} = e^{2qU} \frac{\delta_X L}{2q}, \quad (2.9)$$

in agreement with Eq. (31) of [1] for the field that gets a *vev*. In the remainder of this subsection,  $A \neq A_0$  as  $\Phi^{A_0}$  has been “eaten” by the  $U(1)_X$  superfield.

From the contributions (2.7) to the Kähler potential, the *Einstein condition* (defined by Eq. (A.2) and the associated discussion in Appendix A) is not satisfied in the primed basis. For instance, (2.9) was already shown to lead to an  $O(U)$  violation of the Einstein condition in Eq. (44) of [1]. It therefore becomes necessary to make field redefinitions that involve  $L$  in order to repair this situation. To preserve the linearity of  $L$ , we accomplish this by a Weyl transformation. The details of this method are given in Appendix B. There we also show that the identity (1.8) assures the simultaneous elimination of linear couplings to the heavy vector multiplet.

To proceed, it is convenient to work temporarily with the *modular invariant* real superfields

$$X_A = e^{G'^A} \left( \frac{\delta_X}{2q} \right)^{q_A/q} |\Phi'^A|^2. \quad (2.10)$$

Since  $A \neq A_0$ , these fields all have vanishing *vev*'s:  $\langle X_A \rangle = 0$ . We will exploit this, together with  $\langle U \rangle = 0$ , to make expansions in  $U$  and  $X_A$ , referred to collectively as  $\Delta_A$  in the notation of Appendix B. The redefinition (2.10) is a matter of “tidy bookkeeping,” since it allows for coefficient functionals in Appendix B that are functionals only of  $L$  and not functionals of  $g^I$ . It also ensures that the expansions made there are modular invariant.

The Kähler potential (2.1) then becomes

$$K = \tilde{k}(L) + G + \frac{\delta_X L}{2q} (e^{2qU} - 1) + \sum_{A \neq A_0} L^{q_A/q} X_A e^{2q_A U}. \quad (2.11)$$

Furthermore in the basis  $(L, g^I, \Delta)$  the functional in Eq. (1.3) becomes

$$\tilde{L} = E \left[ -3 + 2LS(L, U, g^I) \right], \quad S(L, U, g^I) = \tilde{s}(L) + \frac{1}{2}\tilde{G} - \frac{\delta_X}{2}U, \quad (2.12)$$

where  $S$  is the functional described in Appendix A and for convenience we define

$$\tilde{s}(L) = s(L) - \frac{\delta_X}{4q} \ln \frac{\delta_X L}{2q}, \quad (2.13)$$

$$\tilde{G} = bG + \frac{\delta_X}{2q} G_q = \sum_I \left( b + \frac{\delta_X}{2q} q^I \right) g^I. \quad (2.14)$$

We note that  $\tilde{k}$  and  $\tilde{s}$  are identically related by the constraint (B.3).

We next apply the results of Appendix B. Note that (2.11) contains terms up to only linear order in  $X_A$ ; thus the expansion (B.1) simplifies to a power series in  $U$ :

$$K = \tilde{k}(L) + G + k^U(L)U + \frac{1}{2}k^{UU}(L)U^2 + \sum_{A \neq A_0} X_A [k^A(L) + k^{UA}(L)U] + O(\Delta^3) \quad (2.15)$$

with coefficients

$$k^U(L) = \delta_X L, \quad k^A(L) = L^{q_A/q}, \quad k^{UU}(L) = 2q\delta_X L, \quad k^{UA}(L) = 2q_A L^{q_A/q}. \quad (2.16)$$

A nice simplification also occurs for  $S$ : we have no  $X_A$  terms and only a linear term in  $U$ , with  $s^U(L) = -\delta_X/2$  in the notation of (B.2).

The parameters of the Weyl transformation to linear order are given by

$$\alpha^U(\hat{L}) = \alpha_0(\hat{L}), \quad \alpha^A(\hat{L}) = \frac{q_A}{\delta_X q} \alpha_0(\hat{L}) \hat{L}^{(q_A-q)/q}. \quad (2.17)$$

The leading coefficients of the shifted Kähler potential  $\hat{K}$  and the functional  $\hat{S}$  are given by

$$\begin{aligned} \hat{K}^U &= \hat{S}^U = 0, & \hat{K}^{UU}(\hat{L}) + 2\hat{L}\hat{S}^{UU}(\hat{L}) &= 2q\delta_X \hat{L} \left( 1 + \frac{\alpha_0(\hat{L})}{6q} \right), \\ \hat{K}^A(\hat{L}) &= \left( 1 - \frac{q_A}{q} \right) \hat{L}^{q_A/q}, & 2\hat{L}\hat{S}^A(\hat{L}) &= \frac{q_A}{q} \hat{L}^{q_A/q}, \\ \hat{K}^{UA}(\hat{L}) + 2\hat{L}\hat{S}^{UA}(\hat{L}) &= 2q_A \hat{L}^{q_A/q} \left( 1 + \frac{\alpha_0(\hat{L})}{6q} \right), \\ \hat{K}^{AB}(\hat{L}) + 2\hat{L}\hat{S}^{AB}(\hat{L}) &= \frac{q_A q_B \alpha_0(\hat{L})}{3q^2 \delta_X \hat{L}} \hat{L}^{(q_A+q_B)/q}. \end{aligned} \quad (2.18)$$

The vanishing of  $\hat{K}^U$  and  $\hat{S}^U$  is a consequence of the Weyl transformation, and gives the desired result that linear couplings to the heavy multiplet  $U$  be eliminated.

The (naively) worrisome  $U$  to  $X_A$  couplings that arise from  $\hat{K}^{UA}(\hat{L}) + 2\hat{L}\hat{S}^{UA}(\hat{L})$  are an artifact of our “bookkeeping” (2.10). These give  $O(U|\Phi'^A|^2)$  contributions in terms of the elementary superfields. The tree exchange of the heavy multiplet  $U$  will give  $O(|\Phi'^A|^2|\Phi'^B|^2)$  effective terms involving space-time derivatives or auxiliary components. Since  $\langle \Phi'^A \rangle \equiv 0$  and auxiliary component *vev*'s are of order of the gravitino mass, these are highly suppressed interaction terms that are negligible for the purposes of our considerations.

Now we examine the content of the  $U(1)_X$  field strength in the new superfield basis. The situation is very similar to that which appeared in our previous article [1]. Once we make

the Weyl transformation, spinorial derivatives are covariant with respect to the new Kähler potential  $\hat{K}$ , and the chiral auxiliary superfield  $\hat{R}$  is also defined with respect the new  $\hat{K}$ . After the Weyl transformation we have  $\mathcal{W}_{V'} \rightarrow \widehat{\mathcal{W}}_{V'}$  wherever the  $U(1)_X$  chiral field strength appeared before, such as in the modified linearity conditions (1.1) or the one-loop effective terms (1.9) and (1.11). However, the redefinition (2.6) now leads to a reinterpretation of this quantity:

$$\begin{aligned}\widehat{\mathcal{W}}_{V'}^\alpha &= -\frac{1}{4}(\hat{\mathcal{D}}^2 - 8\hat{R})\hat{\mathcal{D}}^\alpha V' \\ &= \widehat{\mathcal{W}}_U^\alpha - \frac{1}{8q}(\hat{\mathcal{D}}^2 - 8\hat{R})\hat{\mathcal{D}}^\alpha \left( \ln \frac{\delta_X \hat{L}}{2q} - G_q \right) - \frac{1}{24q}(\hat{\mathcal{D}}^2 - 8\hat{R})\hat{\mathcal{D}}^\alpha \Delta k(\hat{L}, \Delta).\end{aligned}\quad (2.19)$$

In particular, the  $\Delta_A \equiv U$  term in  $\Delta k(\hat{L}, \Delta)$  gives additional contributions to the chiral field strength for the new vector multiplet  $U$ . At leading order in the small quantities  $\Delta_A$ , the correction yields

$$\widehat{\mathcal{W}}_{V'}^\alpha = \left( 1 + \frac{\alpha^U}{6q} \right) \widehat{\mathcal{W}}_U^\alpha + \dots \quad (2.20)$$

Taking into account (2.17), this result is in agreement with Eq. (79) of our previous work [1]. A great many other terms arise from the remainder of (2.19). However, these yield higher order terms in the effective Lagrangian and are negligible at the order of analysis taken up here.

## 2.2 Multiple Vevs

Here the situation is generically more complicated and it is consequently more difficult to decompose the superfields into heavy and light subsets. We first go to a *quasi-unitary gauge*, where the meaning of this term will be made clear in what follows. Define

$$\langle e^{G^A} |\Phi^A|^2 \rangle = |C_A|^2, \quad (2.21)$$

where  $C_A$  is a complex constant. Writing, for  $C_A \neq 0$ ,

$$\Phi^A = C_A e^{\Theta^A}, \quad \Sigma^A = \Theta^A + \bar{\Theta}^A + G^A, \quad \langle \Sigma^A \rangle = 0, \quad (2.22)$$

we go to quasi-unitary gauge by making a gauge transformation that eliminates the eaten chiral multiplet:

$$\begin{aligned}V' &= V + \Theta + \bar{\Theta}, \quad \Phi'^A = e^{-2q_A \Theta} \Phi^A, \\ \Theta &= \frac{1}{2Q} \sum_A q_A B_A \Theta^A, \quad Q = \sum_A q_A^2 B_A.\end{aligned}\quad (2.23)$$

The coefficients  $B_A$  are real constants.  $\Theta$  drops out of the Lagrangian written in terms of  $V', \Phi'$ . If  $C_A \neq 0$  for  $A = 1, \dots, n$  we have for  $A \leq n$

$$\Phi'^A = C_A e^{\Theta'^A}, \quad \Theta'^A = \Theta^A - 2q_A \Theta. \quad (2.24)$$

Only  $n - 1$  of the chiral fields  $\Theta'^A$  are linearly independent:

$$\sum_A q_A B_A \Theta'^A = 0. \quad (2.25)$$

$\Theta$  is chiral but not modular invariant;  $\Phi'^A$  is chiral with modular weight

$$q'_I = q_I^A - 2q_A q_I, \quad q_I = \frac{1}{2Q} \sum_A q_A B_A q_I^A. \quad (2.26)$$

To obtain a modular invariant vector field we set

$$\begin{aligned} U' &= V' + G_X = V + \Sigma, \quad \Sigma = \Theta + \bar{\Theta} + G_X, \\ G_X &= \frac{1}{2Q} \sum_A q_A B_A G^A. \end{aligned} \quad (2.27)$$

$U', \Sigma$  and the matter field contributions to the Kähler potential

$$K(\Phi) = \sum_A K_{(A)}, \quad K_{(A \leq n)} = |C_A|^2 e^{\Sigma'^A + 2q_A U'}, \quad (2.28)$$

are modular invariant. The real fields

$$\Sigma'^A = \Theta'^A + \bar{\Theta}'^A + G'^A, \quad G'^A = G^A - 2q_A G_X, \quad (2.29)$$

have vanishing *vev*'s and satisfy

$$\sum_A q_A B_A \Sigma'^A = 0. \quad (2.30)$$

Finally we make a modular invariant shift in the vector field:

$$U' = U + h(L) + \sum_A b_A(L) \Sigma'^A, \quad \langle U \rangle = 0, \quad (2.31)$$

where  $h, b_A$  are functionals of  $L$  to be determined. We require the D-term *vev* to vanish:

$$\sum_A \left\langle q_A e^{G'^A} |\Phi'^A|^2 e^{2q_A [U + \sum_B b_B(L) \Sigma'^B]} \right\rangle e^{2q_A h(L)} = \frac{\delta_X}{2} L, \quad (2.32)$$

giving an equation for the functional  $h$ :

$$\sum_A q_A |C_A|^2 e^{2q_A h(L)} = \frac{\delta_X}{2} L. \quad (2.33)$$

Now

$$\begin{aligned} k(L) &\rightarrow \tilde{k}(L) = k(L) + \delta k(L), \quad \delta k(L) = \langle K(\Phi) \rangle = \sum_A |C_A|^2 e^{2q_A h(L)}, \\ 2Ls(L) &\rightarrow 2L\tilde{s}(L) = 2Ls(L) + 2L\delta s(L), \quad 2L\delta s(L) = -\delta_X Lh(L). \end{aligned} \quad (2.34)$$

We have using (2.33)

$$\frac{\partial}{\partial L} \delta k = 2h'(L) \sum_A q_A |C_A|^2 e^{2q_A h(L)} = L\delta_X h'(L) = -2L \frac{\partial}{\partial L} \delta s \quad (2.35)$$

so from (1.5) the Einstein condition (A.2) is satisfied for  $U = \Sigma'^A = 0$ .

Now the full Kähler potential is

$$\begin{aligned} K &= k + G + K(\Phi), \\ K(\Phi) &= \sum_A e^{G'^A + 2q_A [U + h(L) + \sum_B b_B \Sigma'^B]} |\Phi'^A|^2 \\ &= \delta k(L) + \sum_A e^{2q_A h} |C_A|^2 \left[ 2q_A U + \left( \Sigma'^A + 2q_A \sum_B b_B \Sigma'^B \right) (1 + 2q_A U) \right] \\ &\quad + O(U^2, \Sigma^2, |\Phi^{A>n}|^2) \\ &= \delta k(L) + L\delta_X U + \sum_A \Sigma'^A \left[ |C_A|^2 e^{2q_A h} (1 + 2q_A U) + b_A \delta_X (L + U/h') \right] \\ &\quad + O(U^2, \Sigma^2, |\Phi^{A>n}|^2), \end{aligned} \quad (2.36)$$

where we used the relation

$$4h'(L) \sum_A q_A^2 |C_A|^2 e^{2q_A h(L)} = \delta_X \quad (2.37)$$

that follows from  $L$ -differentiation of (2.33). The physical, uneaten chiral supermultiplets  $\Theta'^A$  do not mix with the vector field  $U$ . Thus we require

$$2q_A |C_A|^2 e^{2q_A h(L)} + b_A(L) \delta_X / h'(L) = f(L) q_A B_A, \quad (2.38)$$

which eliminates the  $O(\Sigma U)$  terms in (2.36) by virtue of the condition (2.30). We now have

$$\begin{aligned} K &= \tilde{k} + G + L\delta_X U + \sum_A \Sigma'^A |C_A|^2 e^{2q_A h} (1 - 2q_A Lh') + O(U^2, \Sigma^2, |\Phi^{A>n}|^2), \\ S &= \tilde{s} + \frac{1}{2} \tilde{G} - \frac{1}{2} \delta_X \left[ U + h + \sum_A b_A \Sigma'^A \right] \\ &= \tilde{s} + \frac{1}{2} \tilde{G} - \frac{1}{2} \delta_X (U + h) + h' \sum_A q_A |C_A|^2 e^{2q_A h} \Sigma'^A, \\ \tilde{G} &= bG + \delta_X G_X. \end{aligned} \quad (2.39)$$

As before, we perform a Weyl transformation to put the Einstein term in canonical form, replacing the functionals  $K(L, M), S(L, M)$  by  $\hat{K}(\hat{L}, M), \hat{S}(\hat{L}, M)$ . Identifying  $\Delta_A = \Sigma'^A$  in the notation of Appendix B, we have

$$\begin{aligned} k^A &= |C_A|^2 e^{2q_A h} (1 - 2q_A L h') = |C_A|^2 e^{2q_A h} - 2L s^A, \quad k'^A = -2L s'^A, \\ 0 &= \frac{\alpha_0}{\delta_X} (k'^A + 2L s'^A) = \alpha^A = k^{AU} = s^{AU} \\ &= \hat{K}^{AU} + 2\hat{L}\hat{S}^{AU} = k^{AU} + 2\hat{L}s^{AU} + \frac{\delta_X \hat{L}}{3\alpha_0} \alpha^A \alpha^U. \end{aligned} \quad (2.40)$$

Therefore there is no  $U, \Sigma'$  mixing in the effective Kähler potential [7, 9]  $\tilde{K} = \hat{K} + 2\hat{L}\hat{S}$  in the new Weyl basis. The terms linear in  $U$  can be eliminated as before, and further Weyl transformations can be made involving  $\Sigma^2, \Phi^2, U^2$  to get a canonical Einstein term up to quadratic terms, as in the preceding subsection.

To see the relation of our quasi-unitary gauge to the actual unitary gauge that can be determined only when the dilaton and moduli *vev*'s are fixed by supersymmetry breaking, consider the relation between  $U$  and the original vector field  $V$  in terms of the original fields  $\Sigma^A$ .

$$\begin{aligned} h + U &= V + \hat{\Sigma}, \quad \hat{\Sigma} = \Sigma - \sum_B b_B(L) \Sigma'^B = \sum_B f_B(L) \Sigma^B, \\ f_B(L) &= \frac{1}{2Q} q_B B_B \left[ 1 + 2 \sum_A q_A b_A(L) \right] - b_B = \frac{q_B x^B(L)}{2 \sum_C q_C^2 x^C(L)}, \\ x^A(L) &= |C_A|^2 e^{2q_A h(L)}. \end{aligned} \quad (2.41)$$

If  $L$  is replaced by a constant c-number  $\ell_0$ , this is just the redefinition needed to go to unitary gauge starting with the Kähler potential

$$K(\Phi) = \sum_A x^A(\ell_0) e^{\Sigma^A + 2q_A \tilde{V}}, \quad V = \tilde{V} + h(\ell_0), \quad \langle \tilde{V} \rangle = \langle \Sigma^A \rangle = 0. \quad (2.42)$$

Thus once the dilaton and moduli are stabilized and replaced by their *vev*'s  $\ell_0, t_0^I$  one is automatically in unitary gauge, independently of the choice of the parameters  $B_A$  and the functional  $f(L)$ . This freedom in parameter space is presumably related to the large vacuum degeneracy in the absence of other couplings. In particular one can impose  $b_B(\ell_0) = 0$ ,  $\hat{\Sigma}(\ell_0) = \Sigma$ , by setting  $B_A = 2x^A(\ell_0)/f(\ell_0)$ . This still leaves the functional  $f(L)$  undetermined, but it fixes the effective modular weights (2.26) and the corresponding modification (2.27), (2.39) to the GS term. In the case that  $\hat{\Sigma}(\ell_0) \neq \Sigma$ , a further transformation on the chiral multiplets

$$\Phi''^A = e^{-2q_A [\hat{\Theta}(\ell_0) - \Theta]} \Phi' = e^{-2q_A \hat{\Theta}(\ell_0)} \Phi, \quad \hat{\Theta}(L) = \sum_B f_B(L) \Theta^B \quad (2.43)$$

takes us into true unitary gauge, with the term  $-\delta_X \sum_A b_A(\ell_0) G'^A / 2$  in  $S$  in (2.39) interpreted as an additional correction to the GS term  $\tilde{G}$ .

Applying the results of Appendix B for  $\Delta^A = |\Phi^{A>n}|^2$ , the effective Kähler potential for matter is

$$\begin{aligned} \tilde{K}(\Sigma', \Phi') &= \hat{K}(\Sigma', \Phi') + 2\hat{L}\hat{S}(\Sigma', \Phi') = \sum_{A=1}^n x^A \left[ \Sigma'^A + \frac{1}{2} \left( \Sigma'^A + 2q_A \sum_B b_B \Sigma'^B \right)^2 \right] \\ &\quad + \sum_{A>n} e^{2q_A h + G'^A} |\Phi'^A|^2 + O\left(\Sigma^3, \Sigma|\Phi^{A>n}|^2\right). \end{aligned} \quad (2.44)$$

Expanding around the dilaton vacuum in unitary gauge (with  $U = 0$ ) we have

$$\begin{aligned} \hat{L} &= \ell_0 + \hat{\ell}, \quad b_A = O(\hat{\ell}), \\ x^A &= x^A(\ell_0) + 2q_A h' x^A(\ell_0) \hat{\ell} + O(\hat{\ell}^2), \quad \sum_A q_A x^A \Sigma'^A = O(\hat{\ell} \Sigma'), \\ \tilde{K}(\Sigma', \Phi') &= \sum_{A=1}^n x^A(\ell_0) \left[ \Sigma'^A + \frac{1}{2} (\Sigma'^A)^2 \right] \\ &\quad + \sum_{A>n} e^{2q_A h + G'^A} |\Phi'^A|^2 + O\left(\Sigma^3, \Sigma^2 \hat{\ell}, \Sigma \hat{\ell}^2, \hat{\ell} \Sigma |\Phi^{A>n}|^2\right). \end{aligned} \quad (2.45)$$

There is no kinetic mixing of the dilaton with the *D-moduli*.<sup>2</sup> The only effect of the term linear in  $\Sigma'$  is a slight modification of the Kähler potential for the moduli:

$$\delta G_{I\bar{J}} = \sum_{A=1}^n x^A(\ell_0) G'^A_{I\bar{J}}. \quad (2.46)$$

Once the T-moduli are fixed at their *vev's*  $t_0^I$ , chiral D-moduli can be defined as

$$D^A = \hat{\Theta}'^A + \frac{1}{2} G'^A_I(t_0^I) \hat{t}^I, \quad \Sigma'^A = D^A + \bar{D}^A + O\left([\hat{t}^I/t_0^I]^2\right), \quad (2.47)$$

where  $t^I = t_0^I + \hat{t}^I$ . When supersymmetry breaking is included [19] via the condensation model of [20],  $t_0^I = 1$  in reduced Planck units. The  $D^A$  remain massless [18, 17] in the absence of superpotential couplings.

The  $U$  mass is given by

$$m_U^2 = \frac{g_X^2}{2} [\hat{K}^{UU}(\ell_0) + 2\ell_0 \hat{S}^{UU}(\ell_0)] = \frac{\delta_X g_X^2}{2} \left\langle \left( \frac{1}{h'} + \frac{\ell_0 \alpha_0}{3} \right) \right\rangle, \quad (2.48)$$

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<sup>2</sup>The massless modes associated with flat directions of the D-term part of the scalar potential were referred to by this term in our previous work [17]. Such moduli are a generic feature of supersymmetric field theories [18].

where  $g_X$  is the effective  $U(1)_X$  gauge coupling constant that will be made explicit below, and  $\alpha_0$  is defined in (B.18). This reduces to the result found in [1] for the case of a single *vev* with  $h' = 1/2qL$ .

To determine  $g_X$  we start with the Yang Mills Lagrangian

$$\mathcal{L}_{YM} = -\frac{1}{8g^2} \int d^4\theta \frac{E}{R} (\mathcal{W}\mathcal{W})_V, \quad \mathcal{W}_V = -\frac{1}{4}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}V, \quad (2.49)$$

where  $g$  is the renormalized  $U(1)_X$  coupling constant at the  $U$  mass scale:  $g^{-2} = s(\ell_0) + \delta_g$ , with  $\delta_g$  containing corrections from  $\mathcal{L}_Q + \int d^4\theta E\bar{L}\tilde{G}$ . As noted in [1], the shift (2.34) in  $s(\ell_0)$  is canceled by the shift in  $-\delta_X \int d^4\theta ELV$  due to the shift  $h(\ell_0)$  in  $V$ . However as discussed in [1] we have to include the various field redefinitions made in extracting the Yang-Mills Lagrangian for  $U$

$$\mathcal{L}_{YM} = -\frac{1}{8g_X^2} \int d^4\theta \frac{E}{R} (\mathcal{W}\mathcal{W})_U, \quad \mathcal{W}_U = -\frac{1}{4}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}U. \quad (2.50)$$

We have

$$\begin{aligned} V &= U + h(L) + \dots = U + h(e^{\Delta K/3}\hat{L}) + \dots \\ &= U \left[ 1 + h'(\ell_0) \frac{\ell_0 \alpha_0(\ell_0)}{3} \right] + h(\hat{L}) + \dots, \end{aligned} \quad (2.51)$$

where the ellipses represent higher order terms that are negligible for our purposes. This gives, following (2.19),

$$(\mathcal{W}\mathcal{W})_V = \left\langle \left( 1 + h' \frac{\hat{L}\alpha_0}{3} \right)^2 \right\rangle (\mathcal{W}\mathcal{W})_U + \dots \quad (2.52)$$

Thus

$$g_X = \left\langle \frac{3g}{3 + h'\alpha_0\hat{L}} \right\rangle, \quad m_U^2 = \left\langle \frac{3g^2\delta_X}{h'(6 + 2h'\alpha_0\hat{L})} \right\rangle, \quad (2.53)$$

which again reduces to the result of [1] in the case of a single *vev*. As discussed in [1] and the previous subsection, the operator (2.49) contains additional, higher dimensional operators, some of which are linear in  $U$ . When  $U$  is integrated out this leads to new operators of very high dimension in the effective Lagrangian. In the multi-*vev* case there are additional operators due to the presence of the term  $\sum_B b_B \Sigma^B$  in (2.41). These contributions will be of yet higher dimension since  $\langle b_B \rangle = \langle \Sigma^B \rangle = 0$ .

### 3 Generalized Gauge Group

Next we consider the case of  $m$   $U(1)$ 's that are broken by  $n$  scalar *vev*'s. In this case supersymmetry is unbroken provided ( $\delta_{aX} = 0$  for  $a \neq X$ ):

$$\sum_{A=1}^n x^A q_A^a = \frac{\delta_X \ell}{2} \delta_{aX}, \quad a = 1, \dots, m, \quad x^A(\ell) = \langle K_{(A)} \rangle_\ell, \quad (3.1)$$

which has a solution provided  $n \geq m$  with  $m$  of the  $n$  vectors  $q_a^A$  linearly independent. As usual, we will promote (3.1) to a superfield relation. The two subsections below closely parallel those in Section 2 for a single  $U(1)$ .

#### 3.1 Minimal Scalar Vevs

We define a “minimal” set of scalar *vev*'s to mean a minimal set of  $n = m$   $x$ 's that satisfy (3.1). In this case the physical spectrum is just  $m$  massive vector superfields, and it is straightforward to go to unitary gauge. For  $n = m$  (3.1) has a unique inverse:

$$x^A(\ell) = \frac{\delta_X \ell}{2} Q_X^A, \quad x^A(L) = \frac{\delta_X L}{2} Q_X^A, \quad (3.2)$$

where the matrix  $Q_a^A$  is the inverse of  $q_A^a$ :

$$\sum_a Q_a^A q_B^a = \delta_B^A, \quad \sum_A q_A^a Q_b^A = \delta_b^a. \quad (3.3)$$

If  $\langle \Phi^A \rangle \neq 0$  we can write

$$\Phi^A = \xi_A^{\frac{1}{2}} e^{\Theta^A}, \quad K_{(A)} = |\Phi^A|^2 e^{G^A + 2 \sum_a q_A^a V_a} = \xi_A e^{\Theta^A + \bar{\Theta}^A + G^A + 2 \sum_a q_A^a V_a}, \quad (3.4)$$

where  $\xi_A$  is a positive real constant. Then following the procedure of section 2.1 we make a gauge transformation and a sequence of field redefinitions:

$$\begin{aligned} U'_a &= V'_a + G_a = V_a + \Theta_a + \bar{\Theta}_a + G_a = U_a + h_a(L), \quad \langle U_a \rangle \equiv 0, \\ Y_a &= \frac{1}{2} \sum_A Q_a^A Y^A, \quad Y^A = 2 \sum_a q_A^a Y_a, \quad Y = \Theta, G, \end{aligned} \quad (3.5)$$

giving

$$K_{(A)} = \xi_A e^{2 \sum_a q_A^a U'_a} = \xi_A e^{2 \sum_a q_A^a [U_a + h_a(L)]} = x^A(L) e^{2 \sum_a q_A^a U_a}. \quad (3.6)$$

We use (3.3) to solve for the functionals  $h_a(L)$ :

$$h_a(L) = \frac{1}{2} \sum_A Q_a^A \ln(x^A / \xi_A) = \frac{1}{2} \sum_A Q_a^A \ln(\delta_X Q_X^A L / 2\xi_A). \quad (3.7)$$

The constant parameters  $\xi_A$  have no physical significance and just reflect the fact that we can always make global  $U(1)$  transformations with constant parameters. They drop out of

$$\delta k(L) = \sum_A K_{(A)} \Big|_{U_a=0} = \frac{L}{2} \sum_A \delta_X Q_X^A, \quad (3.8)$$

and we have

$$\begin{aligned} \delta s(L) &= \frac{1}{2} \sum_A Q_a^A \ln(\delta_X Q_X^A L / 2\xi_A) = \sum_A Q_a^A \left[ \ln L + \ln(\delta_X Q_X^A / 2\xi_A) \right], \\ \int d^4\theta (L + \Omega) \delta s(L) &= \frac{1}{2} \sum_A Q_a^A \int d^4\theta \ln L (L + \Omega), \end{aligned} \quad (3.9)$$

by the linearity conditions. So the  $\xi_A$  drop out of  $\mathcal{L}_{GS} + \mathcal{L}_Q$ ; a natural choice is  $\xi_A = \delta_X Q_X^A / 2$ . As in the case of a single  $U(1)_X$ , the modified functionals

$$\tilde{k}(L) = k(L) + \delta k(L), \quad \tilde{s}(L) = s(L) + \delta s(L), \quad (3.10)$$

satisfy the Einstein condition (1.5):

$$\tilde{k}'(L) + 2L\tilde{s}'(L) = \delta k'(L) + 2L\delta s'(L) = 0, \quad (3.11)$$

and we have, instead of (2.14),

$$\tilde{G} = bG + \frac{\delta_X}{2} \sum_A Q_X^A G^A = \sum_I g^I \left( b + \frac{\delta_X}{2} \sum_A Q_X^A q_I^A \right). \quad (3.12)$$

Following section 2.1 we perform a Weyl transformation such that in the transformed basis (B.9) of Appendix B is satisfied. As before, this eliminates the leading order terms linear in  $U$ . The quadratic term determines the vector boson mass matrix; as in Sections 2.2 we have to take into account the modification of the effective coupling constant  $g_U$ , which is now a coupling matrix, that is generated by the various field redefinitions. We have

$$\begin{aligned} V_a &= U_a + h_a(L) + \dots = U_a + h_a(e^{\Delta K/3} \hat{L}) + \dots \\ &= U_a + h_a(\hat{L}) + h'_a \frac{\alpha_0}{3} \hat{L} U_X + \dots = U_a + h_a(\hat{L}) + \frac{\alpha_0}{6} \sum_B Q_a^B U_X + \dots \\ &= \mu_{ab} U^b + h_a(\hat{L}) + \dots, \\ \sum_a (\mathcal{W}_V^a)^2 &= \sum_{a,b,c} \mu_{ab} \mu_{ac} \mathcal{W}_U^b \mathcal{W}_U^c + \dots = \sum_d (\mathcal{W}^d)^2 + \dots, \end{aligned} \quad (3.13)$$

$$\mathcal{W}_U^b = \nu^{bd} \mathcal{W}_d, \quad \sum_b \mu_{ab} \nu^{bd} = \delta_a^c,$$

$$\mu_{ab} = \delta_{ab} + Q_a \delta_{bX}, \quad \nu^{bd} = \delta_{bd} - \frac{Q_b \delta_{dX}}{1 + Q_X}, \quad Q_a = \frac{\alpha_0}{6} \sum_B Q_a^B. \quad (3.14)$$

The canonically normalized fields are  $g_a^{-1}\mathcal{W}_a$ , with  $g_a$  the renormalized  $U(1)_a$  coupling constant at the  $U$  mass scale. The squared mass matrix  $m_{ab}^2$  is given by

$$\begin{aligned}(g_a g_b)^{-1} m_{ab}^2 \mathcal{W}^a \mathcal{W}^b &= \frac{1}{2} \hat{K}_{cd} \mathcal{W}_U^c \mathcal{W}_U^d = \nu^{ca} \frac{1}{2} \hat{K}_{cd} \nu^{db} \mathcal{W}_a \mathcal{W}_b, \quad \hat{K}_{cd} = k^{cd} + \frac{\alpha_0}{3} \delta_X \hat{L} \delta_{cX} \delta_{dX}, \\ k^{cd} &= 4 \sum_A q_A^c q_A^d k^A = 2 \sum_A q_A^c q_A^d Q_X^A \delta_X \hat{L}.\end{aligned}\tag{3.15}$$

Putting everything together gives

$$m_{ab}^2 = \frac{g_a g_b}{2} \left\langle k_{ab} - \frac{\delta_{aX} \delta_{bX} \alpha_0 \delta_X \hat{L}}{3(1 + Q_X)} \right\rangle, \tag{3.16}$$

which reduces to the result of [1] in the case of only  $U(1)_X$  with just one scalar *vev*:  $k_{ab} = 2q \delta_X \hat{L}$ ,  $Q_X = \alpha_0/6q$ . As before, the ellipses in (3.13) represent terms that generate higher dimension operators in the effective theory below the  $U$  mass scale.

### 3.2 Nonminimal Case

This is a straightforward generalization of the case discussed in Section 2.2. We first go to quasi-unitary gauge by setting

$$\begin{aligned}U'_a &= V_a + \Sigma_a, \quad \Phi'^A = e^{-2 \sum_a q_A^a \Theta_a} \Phi^A, \quad \Sigma_a = \Theta_a + \bar{\Theta}_a + G_a, \\ \Theta_a &= \frac{1}{2} \sum_{A,b} q_A^b M_{ba}^{-1} B_A \Theta^A, \quad G_a = \frac{1}{2} \sum_{A,b} q_A^b M_{ba}^{-1} B_A G^A, \\ M_{ab} &= \sum_A q_A^a q_A^b B_A, \quad a, b = 1 \dots m.\end{aligned}\tag{3.17}$$

$\Phi'^A$  is chiral with modular weight

$$q_I'^A = q_I^A - 2 \sum_a q_A^a q_I^a, \quad q_I^a = \frac{1}{2} \sum_{A,b} q_A^b M_{ba}^{-1} B_A q_I^A, \tag{3.18}$$

and  $U_a$  is modular invariant. Without the  $G_a$ -terms, (3.17) is a gauge transformation;  $\Theta_a$  drops out of the Lagrangian. If  $C_A \neq 0$  for  $A = 1, \dots, n$ , with  $C_A$  defined as in (2.21),

$$\Phi'^A = C_A e^{\Theta'^A}, \quad \Theta'^A = \Theta^A - 2 \sum_a q_A^a \Theta_a. \tag{3.19}$$

Only  $n - m$  of the chiral fields  $\Theta'^A$  are linearly independent, where  $m$  is the number of linearly independent  $\Theta_a$ :

$$\sum_A q_A^a B_A \Theta'^A = 0 \quad \forall a. \tag{3.20}$$

The real fields  $\Sigma'^A = \Theta'^A + \bar{\Theta}'^A + G'^A$  have vanishing *vev*'s and satisfy

$$\sum_A q_A^a B_A \Sigma'^A = 0. \quad (3.21)$$

Now set

$$U'_a = U_a + h_a(L) + \sum_B b_{aB} \Sigma'^B, \quad \langle U_a \rangle = 0. \quad (3.22)$$

Using the notation introduced above, we have

$$\begin{aligned} \delta k(L) &= \sum_A |C_A|^2 \prod_a e^{2q_A^a h_a}, \\ \frac{\partial}{\partial L} \delta k(L) &= 2 \sum_A |C_A|^2 \sum_b q_b^A h'_b \prod_a e^{2q_A^a h_a} \\ &= 2 \sum_b h'_b \sum_A |C_A|^2 q_A^b \prod_a e^{2q_A^a h_a} = h'_X \delta_X L, \end{aligned} \quad (3.23)$$

because the vanishing of D-terms requires

$$\sum_A |C_A|^2 q_A^b \prod_a e^{2q_A^a h_a} = \delta_{bX} \frac{\delta_X}{2} L. \quad (3.24)$$

In addition

$$2L\delta s = -\delta_X L h_X, \quad 2L \frac{\partial}{\partial L} \delta s = -\delta_X L h'_X = -\frac{\partial}{\partial L} \delta k, \quad (3.25)$$

and the Einstein condition (A.2) is again satisfied for  $U = \Sigma = 0$ . The Kähler potential for matter is

$$\begin{aligned} K(\Phi) &= \sum_A e^{G'^A + 2 \sum_a q_A^a [U_a + h(L)_a + \sum_B b_{aB} \Sigma'^B]} |\Phi'^A|^2 \\ &= \delta k(L) + L \delta_X U_X + \sum_A \Sigma'^A (x^A + b_{XA} \delta_X L) \\ &\quad + 2 \sum_{A,a} \Sigma'^A U_a \left( q_A^a x^A + 2 \sum_{B,b} b_{bA} x^B q_B^b q_B^a \right) + O(U^2, \Sigma^2, |\Phi^{A>n}|^2), \\ x^A &= |C_A|^2 e^{2 \sum_b q_A^b h_b}. \end{aligned} \quad (3.26)$$

To eliminate  $U, \Sigma'$  mixing we impose

$$\begin{aligned} f_a(L) q_A^a B_A &= q_A^a x^A + 2 \sum_{B,b} b_{bA} x^B q_B^b q_B^a, \quad b_{aA} = \frac{1}{2} \sum_b q_A^b N_{ab}^{-1} (f_b B_A - x^A), \\ N_{ab} &= \sum_B x^B q_B^a q_B^b, \quad \sum_A b_{aA} \Sigma'^A = -\frac{1}{2} \sum_b N_{ab}^{-1} x^A q_A^b \Sigma'^A \equiv \sum_A \hat{b}_{aA} \Sigma'^A, \end{aligned} \quad (3.27)$$

and we proceed as before with a Weyl transformation to make the Einstein term canonical to quadratic order in  $U, \Sigma, \Phi^{A>n}$ . We now have

$$\begin{aligned} K &= \tilde{k} + G + L\delta_X U_X + \sum_A k^A \Sigma'^A + O(U^2, \Sigma^2, |\Phi^{A>n}|^2), \\ S &= \tilde{s} + \frac{1}{2}\tilde{G} + \frac{1}{2}\delta_X \left( U_X + \sum_A b_{XA} \Sigma'^A \right), \quad \tilde{G} = bG + \delta_X G_X, \\ k^A &= x^A + \hat{b}_{XA} \delta_X L = x^A - 2Ls^A, \quad k'^A = 2x^A \sum_a q_A^a h'_a + \hat{b}_{XA} \delta_X - 2Ls'^A. \end{aligned} \quad (3.28)$$

Differentiation of (3.24) with respect to  $L$  gives

$$2 \sum_a h'_a N_{ab} = \delta_{bX} \frac{\delta_X}{2}, \quad 2h'_a = N_{aX}^{-1} \frac{\delta_X}{2}, \quad \delta_X \hat{b}_{XA} = -\frac{\delta_X}{2} \sum_a N_{aX}^{-1} = -2x^A \sum_a q_A^a h'_a. \quad (3.29)$$

So we have for  $\Delta_A = \Sigma'^A$

$$k'^A + 2Ls'^A = 0 = \alpha^A, \quad (3.30)$$

and as in Section 2.2 the  $U_a$  remain decoupled from the  $\Sigma'^A$  in the Weyl transformed basis, and there are no terms linear in  $U$  in this basis. Following (2.41) we write

$$\begin{aligned} h_a + U_a &= V_a + \hat{\Sigma}_a, \quad \hat{\Sigma}_a = \Sigma_a - \sum_B b_{aB}(L) \Sigma'^B = \sum_B f_{aB}(L) \Sigma^B, \\ f_{aB}(L) &= \frac{1}{2} \sum_b q_B^b B_B \left( M_{ab}^{-1} + 2 \sum_{Ac} q_A^c M_{cb}^{-1} b_{aA} \right) - b_{aB} = \frac{1}{2} \sum_b N_{ab}^{-1} q_B^b x^B. \end{aligned} \quad (3.31)$$

This again is the required redefinition of the vector field at the dilaton vacuum:  $L \rightarrow \ell_0$ , and corresponds to the true unitary gauge provided

$$\langle B_{aA} \rangle = 0, \quad B_A = x^A(\ell_0)/f(\ell_0), \quad f_a(\ell_0) = f(\ell_0), \quad (3.32)$$

which requires  $f_a$  independent of  $a$  up to terms that vanish in the vacuum.

We set the massive  $U_a$ 's to zero to obtain the effective low energy theory. As in Section 2.2 we can expand around the vacuum values of the dilaton and T-moduli to obtain the effective Kähler potential for matter:

$$\begin{aligned} \tilde{K}(\Sigma'^A, \Phi'^A) &= \delta G(T, \bar{T}) + \sum_{A=1}^n x^A(\ell_0) \left( D^A + \bar{D}^A \right)^2 + \sum_{A>n} e^{2q_A h + G'^A} |\Phi'^A|^2 \\ &\quad + O\left(\hat{\phi}^3\right), \end{aligned} \quad (3.33)$$

where  $\hat{\phi}$  is any field with vanishing vev, and we used

$$\sum_A q_A^a x^A \Sigma'^A = O(\hat{\ell} \Sigma'). \quad (3.34)$$

The  $n - m$  massless D-moduli  $D^A$  and the shift  $\delta G_{I\bar{J}}$  in the T-moduli metric are defined as in subsection 2.2.

Finally, to determine the  $U$  mass matrix, we proceed as in the previous subsection:

$$\begin{aligned}
 V_a &= U_a + h_a(L) + \dots = U_a + \left\langle h'_a \frac{\alpha_0}{3} \hat{L} \right\rangle U_X + h_a(\hat{L}) + \dots \\
 &= \mu_{ab} U^b + h_a(\hat{L}) + \dots, \\
 \sum_a (\mathcal{W}_V^a)^2 &= \sum_{a,b,c} \mu_{ab} \mu_{ac} \mathcal{W}_U^b \mathcal{W}_U^c + \dots = \sum_d (\mathcal{W}^d)^2 + \dots, \\
 \mathcal{W}_U^b &= \nu^{bd} \mathcal{W}_d, \quad \sum_b \mu_{ab} \nu^{bd} = \delta_a^c, \\
 \mu_{ab} &= \delta_{ab} + Q_a \delta_{bX}, \quad \nu^{bd} = \delta_{bd} - \frac{Q_b \delta_{dX}}{1 + Q_X}, \quad Q_a = \frac{\alpha_0}{6} h'_a,
 \end{aligned} \tag{3.35}$$

giving

$$m_{ab}^2 = \frac{g_a g_b}{2} \nu^{ca} \hat{K}_{cd} \nu^{db} = \frac{g_a g_b}{2} \left\langle k_{ab} - \frac{\delta_{aX} \delta_{bX} \alpha_0 \delta_X \hat{L}}{3(1 + Q_X)} \right\rangle, \tag{3.36}$$

which reduces to the result (3.16) in the minimal case with  $h'_a = \sum_A Q_a^A / \hat{L}$ . As in Section 2.2, the higher dimension operators implied by the ellipses in (3.35) include terms arising from  $\sum_A b_{aA} \Sigma^A$ .

## 4 Superpotential

Now we want to address linear couplings to heavy fields that may appear in the superpotential, and how these are eliminated by superfield redefinitions. We will also have superpotential terms that give masses to some more chiral multiplets. The *vev*'s have to be in F-flat directions, so the superpotential has to be at most<sup>3</sup> linear in  $\Phi^A$  if  $C_A \neq 0$ . The superpotential terms are of the form

$$\prod_A \Phi^A f(T) \tag{4.1}$$

where  $f(T)$  makes the expression modular covariant. Now  $\prod_A \Phi^A$  is invariant under all the  $U(1)$ 's:  $\sum_A q_A^a = 0$ , so when we make the gauge transformation (3.17) the total modular weight doesn't change and the superpotential remains modular covariant, as can be checked. Now suppose we have a term

$$\Phi_1 \Phi_2 \Phi_3 f(T), \quad C_1 \neq 0. \tag{4.2}$$

---

<sup>3</sup>In actuality, the superpotential can be more than linear in  $\Phi^A$  in terms that are of dimension greater than 3, i.e. in any term that has at least 2 fields with vanishing *vev*'s.

$\Phi_2, \Phi_3$  combine to form a massive supermultiplet. There might also be terms linear in these:

$$\begin{aligned} W &\ni \Phi_1\Phi_2\Phi_3 f(T) + \Phi_2w_3(\Phi) + \Phi_3w_2(\Phi) = \Phi_1\Phi'_2\Phi'_3 f(T) - (\Phi_1 f)^{-1}w_2w_3, \\ \Phi'_i &= \Phi_i + (\Phi_1 f)^{-1}w_i(\Phi). \end{aligned} \quad (4.3)$$

The last term is dimension 4 since  $w^i \sim (\Phi)^2$  and by assumption  $\langle w_i \rangle \neq 0$ , so for most purposes we can drop it and set  $\Phi'_{2,3} = 0$  in the superpotential. However, the last term may be of interest if it generates highly suppressed operators that are otherwise forbidden.

For example suppose

$$w_2 = \lambda_{ij}^2 Q^i Q^j, \quad w_3 = \lambda_{ij}^3 Q^i L^j, \quad (4.4)$$

where  $Q^i$  ( $i = 1, 2, 3$ ) are the three generations of quark doublet superfields and  $L^i$  ( $i = 1, 2, 3$ ) are the lepton doublet superfields. We identify  $\Phi_2 \equiv D^c$ ,  $\Phi_3 \equiv D$ , a gauge-vector pair of color-triplet,  $SU(2)_L$  singlet, chiral superfields. Thus we obtain from (4.3) the effective superpotential operator

$$-(\Phi_1 f)^{-1}w_2w_3 = -(\Phi_1 f)^{-1}\lambda_{ij}^2\lambda_{k\ell}^3 Q^i Q^j Q^k L^\ell \quad (4.5)$$

that mediates nucleon decay. In contrast to the minimal GUT case, the couplings  $\lambda_{ij}^2$  and  $\lambda_{ij}^3$  have no reason to be hierarchically small for the light quark generations, since these are not the Yukawa couplings that give masses to these quarks. Thus, even if the effective vector mass for the  $D, D^c$  pair is of the order of the usual colored Higgs scale, we can exceed proton decay limits by several orders of magnitude. To further address such issues requires a model dependent analysis of the string scale couplings of MSSM supermultiplets to exotic supermultiplets and the flat directions that yield the effective vector mass terms for exotic quarks.

## 5 Conclusions

We have considered the case of several scalar fields, charged under a number of  $U(1)$  factors, acquiring vacuum expectation values due to an anomalous  $U(1)$ . We have demonstrated how to make redefinitions at the superfield level in order to account for tree-level exchange of massive vector superfields in the effective supergravity theory of the light fields in the supersymmetric vacuum phase. Our approach has built upon previous results that we obtained in a more elementary case. We found that the modular weights of light fields are typically shifted from their original values, allowing an interpretation in terms of the preservation of modular invariance in the effective theory. We have addressed the subtleties

in defining unitary gauge, associated in part with the noncanonical Kähler potential that occurs in modular invariant supergravity. Further complications arise from the role of the dilaton as the order parameter in the (most realistic) case where the vacuum is degenerate in  $U(1)$ -charged scalar space (D-moduli space). We have discussed the effective superpotential for the light fields and have noted how proton decay operators may be obtained when the heavy fields are integrated out of the theory at the tree-level.

We still need to include spontaneous symmetry breaking by gaugino condensation in a hidden sector and the related soft supersymmetry breaking phenomenology. Work in this direction will be presented in a future publication [19]. A related issue, that will also be taken up in [19], is the stabilization of the dilaton  $\ell$  in the presence of matter fields with large *vev*'s. In [2] it was noted that all models in the class studied there suffer from a T-moduli mass problem. Stabilization of the T-moduli through an effective theory of gaugino condensation will allow us to address how the moduli masses may change in the presence of a  $U(1)_X$  factor; indeed, we will show that the mass *is* modified and that this may ameliorate the moduli problem discussed in [2]. It also remains to be studied how the required couplings to hidden matter condensates will stabilize the D-moduli. This has been touched on in a previous letter [21], but a full-fledged analysis where tree-level exchange has been taken into account needs to be performed. This too will appear in future work [19]. Finally, it is important to understand how the presence of large *vev*'s could effect other phenomenological aspects of semi-realistic models.

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## Appendices

### A Canonical Einstein Normalization

Of chief concern in our considerations is the maintenance of the canonical normalization for the Einstein term—concurrent to field redefinitions. Therefore we lay out a general prescription for determining the necessary *Einstein condition* from  $\mathcal{L}$  rewritten in a new field

basis.

The relevant part of the Lagrangian is (1.3). We define  $M$  to stand collectively for the fields that are to be regarded as independent of  $L$  in a given basis. We then define the functional  $S$  by the identification

$$\tilde{L} \equiv E[-3 + 2LS(L, M)]. \quad (\text{A.1})$$

The Einstein condition holds provided

$$\left(\frac{\partial K}{\partial L}\right)_M + 2L \left(\frac{\partial S}{\partial L}\right)_M = 0. \quad (\text{A.2})$$

Here the subscripts on parentheses instruct us to hold constant under differentiation the fields denoted collectively by  $M$ .

## B Weyl Transformation

First we fix to unitary gauge such that we have a basis of fields written in terms of  $L$  and modular invariant real superfields  $\Delta_A$  that satisfy  $\langle \Delta_A \rangle = 0$ . For instance in the text the  $\Delta_A$  stand collectively for unitary gauge vector multiplets  $U_a$  with vanishing *vev*'s, the modular invariant composite superfields  $X_A$ , and, for  $\langle X_A \rangle \neq 0$ , the superfields  $\Sigma_A = \ln(X_A/\langle X_A \rangle)$ . We can quite generally write the Kähler potential as a power series in the superfields with vanishing *vev*'s:

$$K(L, g^I, \Delta) = \tilde{k}(L) + G + \sum_A k^A(L)\Delta_A + \frac{1}{2} \sum_{AB} k^{AB}(L)\Delta_A\Delta_B + O(\Delta^3). \quad (\text{B.1})$$

Similarly we can write the functional  $S$  defined in Appendix A as

$$S(L, g^I, \Delta) = \tilde{s}(L) + \frac{1}{2}\tilde{G} + \sum_A s^A(L)\Delta_A + \frac{1}{2} \sum_{AB} s^{AB}(L)\Delta_A\Delta_B + O(\Delta^3). \quad (\text{B.2})$$

The functionals  $G$  and  $\tilde{G}$  are defined in the main text and are independent of  $L$  and the fields  $\Delta_A$ . We also assume the identity, shown in the text to hold,

$$\tilde{k}'(L) + 2L\tilde{s}'(L) = 0. \quad (\text{B.3})$$

To put the Einstein term in canonical form for the terms involving the  $\Delta_A$  fields, we make a Weyl transformation

$$K \equiv \hat{K} + \Delta k \quad (\text{B.4})$$

such that

$$E = e^{-\Delta k/3} \hat{E}, \quad L = e^{\Delta k/3} \hat{L}, \quad (\text{B.5})$$

with  $\Delta k$  an  $O(\Delta)$  functional with assumed form

$$\Delta k(\hat{L}, \Delta) = \sum_A \alpha^A(\hat{L}) \Delta_A + \sum_{AB} \beta^{AB}(\hat{L}) \Delta_A \Delta_B + O(\Delta^3). \quad (\text{B.6})$$

Thus we pass to a new superfield basis  $(\hat{L}, g^I, \Delta)$ . From (A.1) the Einstein condition is restated in terms of a new functional  $\hat{S}(\hat{L}, g^I, \Delta)$  given by the identification

$$2\hat{L}\hat{S}(\hat{L}, g^I, \Delta) \equiv 2\hat{L} S(L, g^I, \Delta) \Big|_{L=e^{\Delta k/3}\hat{L}} + 3(1 - e^{-\Delta k/3}). \quad (\text{B.7})$$

The field redefinition (B.5) assures that the linearity condition for  $L \rightarrow \hat{L}$  is not modified.<sup>4</sup> This can be seen by first leaving  $L$  unconstrained and writing

$$\tilde{L} \rightarrow \tilde{L}' = \tilde{L} - E(S + \bar{S})(L + \Omega), \quad (\text{B.8})$$

where  $\Omega$  is the Chern-Simons superfield:  $(\bar{\mathcal{D}}^2 - 8R)\Omega = \mathcal{W}^\alpha \mathcal{W}_\alpha$ . Under a Weyl transformation:  $E \rightarrow XE = \hat{E}$ ,  $\Omega \rightarrow X^{-1}\Omega = \hat{\Omega}$ ,  $L \rightarrow X^{-1}L = \hat{L}$ ,  $E\Omega = \hat{E}\hat{\Omega}$  and  $EL = \hat{E}\hat{L}$  are Weyl invariant [22]. Then (B.8) takes the same form in terms of the hatted fields, and the equations of motion for the chiral and anti-chiral superfields  $S, \bar{S}$  give the linearity conditions for  $\hat{L}$ .

In the new basis and in terms of the functionals  $\hat{K}$  and  $\hat{S}$  we require (A.2) to have the canonical Einstein term:

$$\left( \frac{\partial \hat{K}}{\partial \hat{L}} \right)_{g^I, \Delta} + 2\hat{L} \left( \frac{\partial \hat{S}}{\partial \hat{L}} \right)_{g^I, \Delta} = 0. \quad (\text{B.9})$$

Using the definitions (B.4) and (B.7) it is straightforward to express (B.9) in terms of the original functionals  $K$  and  $S$ , as well as the Weyl transformation functional  $\Delta k$ :

$$\frac{\partial \hat{K}}{\partial \hat{L}} + 2\hat{L} \frac{\partial \hat{S}}{\partial \hat{L}} = \frac{\partial L}{\partial \hat{L}} [K'(L) + 2\hat{L}S'(L)] - \left( \frac{3}{\hat{L}} - \frac{\partial \Delta k}{\partial \hat{L}} \right) (1 - e^{-\Delta k/3}). \quad (\text{B.10})$$

Here, we use the shorthand notation

$$K'(L) \equiv \frac{\partial K(L, g^I, \Delta)}{\partial L} \Big|_{L=e^{\Delta k/3}\hat{L}} \quad (\text{B.11})$$

---

<sup>4</sup>As discussed in the main text, however, the redefinitions of vector superfields lead to a reinterpretation of the chiral field strengths in the new superfield basis. Similarly, the Chern-Simons superfield  $\Omega$  used here must also be reinterpreted in the new superfield basis.

and similarly for  $S'(L)$ . We also note that

$$\frac{\partial L}{\partial \hat{L}} = \frac{\partial L(\hat{L}, \Delta)}{\partial \hat{L}} = e^{\Delta k/3} \left( 1 + \frac{\hat{L}}{3} \frac{\partial \Delta k}{\partial \hat{L}} \right) \quad (\text{B.12})$$

is easily computed using (B.5).

We next expand (B.10) in powers of  $\Delta$  and demand that it vanish at each order so that (B.9) will hold in the new basis. This involves power series expansion in  $\Delta$  of the  $L$ -dependent coefficients appearing in (B.1) and (B.2) corresponding to the expansion of the quantity

$$K'(L) + 2\hat{L}S'(L) = K'(\hat{L}) + 2\hat{L}S'(\hat{L}) + \hat{L} \left( e^{\Delta k/3} - 1 \right) [K''(\hat{L}) + 2\hat{L}S''(\hat{L})] + O(\Delta^2), \quad (\text{B.13})$$

where we denote here and elsewhere below

$$K'(\hat{L}) \equiv K'(L)|_{L=\hat{L}}, \quad K''(\hat{L}) \equiv K''(L)|_{L=\hat{L}}, \quad \text{etc.} \quad (\text{B.14})$$

Since  $K'(\hat{L}) + 2\hat{L}S'(\hat{L}) = \tilde{k}'(\hat{L}) + 2\hat{L}\tilde{s}'(\hat{L}) = 0$  holds identically with Eq. (B.3), we have that  $K'(L) + 2\hat{L}S'(L)$  is  $O(\Delta)$ . Thus the first nontrivial conditions that arise are at the order of terms linear in  $\Delta$ . We obtain

$$\frac{\partial \hat{K}}{\partial \hat{L}} + 2\hat{L} \frac{\partial \hat{S}}{\partial \hat{L}} = \sum_A \left\{ k'^A(\hat{L}) + 2\hat{L}s'^A(\hat{L}) + \frac{\hat{L}}{3} \left[ \tilde{k}''(\hat{L}) + 2\hat{L}\tilde{s}''(\hat{L}) - \frac{3}{\hat{L}^2} \right] \alpha^A(\hat{L}) \right\} \Delta_A + O(\Delta^2), \quad (\text{B.15})$$

where

$$k'^A(\hat{L}) = \frac{dk^A(L)}{dL} \Big|_{L=\hat{L}}, \quad s'^A(\hat{L}) = \frac{ds^A(L)}{dL} \Big|_{L=\hat{L}}, \quad \text{etc.} \quad (\text{B.16})$$

We must choose each term in the sum of Eq. (B.15) to vanish. It is convenient to rewrite these constraints in the following manner. First note that differentiating Eq. (B.3) and then sending  $L \rightarrow \hat{L}$  yields

$$\hat{L} [\tilde{k}''(\hat{L}) + 2\hat{L}\tilde{s}''(\hat{L})] = -2\hat{L}\tilde{s}'(\hat{L}) = \tilde{k}'(\hat{L}). \quad (\text{B.17})$$

Furthermore we introduce the functional  $\alpha_0(\hat{L})$  defined in our previous article:

$$\alpha_0(\hat{L}) \equiv \frac{3\delta_X \hat{L}}{3 - \hat{L}\tilde{k}'(\hat{L})}. \quad (\text{B.18})$$

Then the right-hand side of (B.15) may be written

$$\frac{\partial \hat{K}}{\partial \hat{L}} + 2\hat{L} \frac{\partial \hat{S}}{\partial \hat{L}} = \sum_A \left[ k'^A(\hat{L}) + 2\hat{L}s'^A(\hat{L}) - \delta_X \frac{\alpha^A(\hat{L})}{\alpha_0(\hat{L})} \right] \Delta_A + O(\Delta^2). \quad (\text{B.19})$$

To have the Einstein condition satisfied to  $O(\Delta)$  we require that the linear coefficients in (B.6) be taken as

$$\alpha^A(\hat{L}) = \frac{1}{\delta_X} \alpha_0(\hat{L}) \left[ k'^A(\hat{L}) + 2\hat{L}s'^A(\hat{L}) \right]. \quad (\text{B.20})$$

Once the  $\hat{L}$ -dependent coefficients in (B.6) have been determined from the requirement (B.9), as has just been done to  $O(\Delta)$ , then the functionals  $\hat{K}(\hat{L}, g^I, \Delta)$  and  $\hat{S}(\hat{L}, g^I, \Delta)$  are completely determined. We define corresponding expansions in the “small” superfields  $\Delta_A$ :

$$\hat{K}(\hat{L}, g^I, \Delta) \equiv \tilde{k}(\hat{L}) + G + \sum_A \hat{K}^A(\hat{L}) \Delta_A + \frac{1}{2} \sum_{AB} \hat{K}^{AB}(\hat{L}) \Delta_A \Delta_B + O(\Delta^3), \quad (\text{B.21})$$

$$\hat{S}(\hat{L}, g^I, \Delta) = \tilde{s}(\hat{L}) + \frac{1}{2} \tilde{G} + \sum_A \hat{S}^A(\hat{L}) \Delta_A + \frac{1}{2} \sum_{AB} \hat{S}^{AB}(\hat{L}) \Delta_A \Delta_B + O(\Delta^3), \quad (\text{B.22})$$

It is useful to determine the  $\hat{L}$ -dependent coefficients to leading orders using the  $O(\Delta)$  results given above, e.g., Eq. (B.20).

Making the necessary expansions of the  $L$ -dependent quantities that appear in (B.1) and (B.2) to get to the basis  $(\hat{L}, g^I, \Delta)$ , we obtain

$$\hat{K}^A(\hat{L}) = \left( \frac{1}{3} \hat{L} \tilde{k}'(\hat{L}) - 1 \right) \alpha^A(\hat{L}) + k^A(\hat{L}), \quad (\text{B.23})$$

$$\begin{aligned} 2\hat{L}\hat{S}^A(\hat{L}) &= \left( 1 + \frac{2}{3} \hat{L}^2 \tilde{s}'(\hat{L}) \right) \alpha^A(\hat{L}) + 2\hat{L}s^A(\hat{L}), \\ &= \left( 1 - \frac{1}{3} \hat{L} \tilde{k}'(\hat{L}) \right) \alpha^A(\hat{L}) + 2\hat{L}s^A(\hat{L}), \end{aligned} \quad (\text{B.24})$$

where we have used (B.3) in the last step. Taking (B.20) and (B.18) into account we rewrite these as

$$\hat{K}^A(\hat{L}) = k^A(\hat{L}) - \hat{L} \left( k'^A(\hat{L}) + 2\hat{L}s'^A(\hat{L}) \right), \quad (\text{B.25})$$

$$2\hat{L}\hat{S}^A(\hat{L}) = 2\hat{L}s^A(\hat{L}) + \hat{L} \left( k'^A(\hat{L}) + 2\hat{L}s'^A(\hat{L}) \right). \quad (\text{B.26})$$

We note the simplicity of the  $O(\Delta)$  contribution to  $\hat{K} + 2\hat{L}\hat{S}$ :

$$\hat{K}^A(\hat{L}) + 2\hat{L}\hat{S}^A(\hat{L}) = k^A(\hat{L}) + 2\hat{L}s^A(\hat{L}). \quad (\text{B.27})$$

Since  $U_a$  is linear in  $V_a$  we have

$$k^{U_a} = k^{V_a} \equiv k^a, \quad s^{U_a} = s^{V_a} \equiv s^a. \quad (\text{B.28})$$

Then it follows from (1.8) that

$$\begin{aligned} k^a(\hat{L}) + 2\hat{L}s^a(\hat{L}) &= 0, \quad \hat{L} \left( k'^a(\hat{L}) + 2\hat{L}s'^a(\hat{L}) \right) = -2\hat{L}^2 s^a(\hat{L}) = k^a(\hat{L}), \\ \hat{K}^a(\hat{L}) &= 2\hat{L}\hat{S}^a(\hat{L}) = 0. \end{aligned} \quad (\text{B.29})$$

Thus the unwanted linear couplings in  $U_a$  are automatically removed by the Weyl transformation.

The  $O(\Delta^2)$  coefficient functionals are more complicated, and involve the functional  $\beta^{AB}(\hat{L})$  that should follow from (B.9) and (B.10) at  $O(\Delta^2)$  but was not explicitly determined above. However, for our purposes we need only the  $O(\Delta^2)$  contribution to  $\hat{K} + 2\hat{L}\hat{S}$ , that as it turns out does not depend on  $\beta^{AB}(\hat{L})$ . We find:

$$\begin{aligned} \hat{K}^{AB} + 2\hat{L}\hat{S}^{AB} &= k^{AB}(\hat{L}) + 2\hat{L}s^{AB}(\hat{L}) + \alpha^A \alpha^B \left[ \frac{\hat{L}^2}{9} (\tilde{k}''(\hat{L}) + 2\hat{L}\tilde{s}''(\hat{L})) - \frac{1}{3} \right] \\ &\quad + \frac{\hat{L}}{3} \left[ (k'^A(\hat{L}) + 2\hat{L}s'^A(\hat{L})) \alpha^B + (k'^B(\hat{L}) + 2\hat{L}s'^B(\hat{L})) \alpha^A \right] \\ &= k^{AB}(\hat{L}) + 2\hat{L}s^{AB}(\hat{L}) + \frac{\delta_X \hat{L}}{3\alpha_0} \alpha^A \alpha^B. \end{aligned} \quad (\text{B.30})$$

In the second step we have exploited (B.3), (B.17), (B.18) and (B.20) to simplify considerably.

In the theory defined by (1.2) quadratic terms in the  $\Delta_A$  appear only through the combination of functionals  $\hat{K} + 2\hat{L}\hat{S}$ . The quantity given in (B.30) is therefore the effective metric for these supermultiplets. Research in progress [19] shows that when a gaugino condensate potential is added, quadratic terms in  $\Delta_A$  appear through  $\hat{K}$  by itself, rather than in the combination  $\hat{K} + 2\hat{L}\hat{S}$ . However, this lone  $\hat{K}$  appears only in terms proportional to the squared condensate  $|\langle \lambda \lambda \rangle|^2$ . We find that any nonvanishing vev's  $\langle \Delta_A \rangle$  are naturally of the same order (since they represent a shift away from the supersymmetric vacuum that was stable in the absence of gaugino condensation), so we don't have to evaluate  $\beta^{AB}$  because it only appears in these negligible terms.

## C Generalized GS Mechanism

In this appendix we address the situation that arises in Type I and Type IIB four-dimensional  $N = 1$  superstring models; for example in [23]. Here the anomaly matching that occurs in the weakly-coupled heterotic models considered above no longer holds and a more general GS term is required. The cancellation of  $U(1)$  anomalies results from couplings to two-forms from the *twisted* closed string sectors, which we will denote  $b_{(A)m n}$ . Since these two-forms are contained in linear multiplets of the underlying theory [24], the generalized GS mechanism can easily be incorporated into the present formalism, as we now describe.

Suppose a modification of the GS counterterm Lagrangian such that

$$\mathcal{L}_{GS} \ni -\frac{1}{2} \sum_{a \in \mathcal{G}_X, A} c_{Aa} B_{(A)}^m v_{(a)m}. \quad (\text{C.1})$$

Here,  $\mathcal{G}_X$  is a product of anomalous  $U(1)$ 's and  $v_{(a)m}$  are the corresponding vector bosons. The one-form  $B_{(A)}^m$  is a gauge-invariant (dual) field strength obtained from coupling the two-form to a combination of Chern-Simons three-forms for each of the simple factors  $\mathcal{G}_a$  in the full gauge group  $\mathcal{G}$ :

$$B_{(A)}^m = \frac{1}{2} \epsilon^{mnpq} \left( \partial_n b_{(A)pq} + \frac{2}{3} \sum_{a \in \mathcal{G}} \tilde{c}_{Aa} \Omega_{(a)npq} \right). \quad (\text{C.2})$$

This is a straightforward generalization of the coupling of a single two-form field strength to Chern-Simons three-forms, as has been described for instance in [22, 10, 9]. It then follows from this definition that

$$\partial_m B_{(A)}^m = \sum_{a \in \mathcal{G}} \tilde{c}_{Aa} (F \cdot \tilde{F})_a. \quad (\text{C.3})$$

With this in mind it is easy to see that under a gauge transformation acting on the anomalous  $U(1)$  vector bosons according to  $v_{(a)m} \rightarrow v_{(a)m} + \partial_m \lambda_{(a)}$ , Eq. (C.1) shifts as (up to a total derivative)

$$\begin{aligned} \delta \mathcal{L}_{GS} &= -\frac{1}{2} \sum_{a \in \mathcal{G}_X, A} c_{Aa} B_{(A)}^m \partial_m \lambda_{(a)} = \frac{1}{2} \sum_{a \in \mathcal{G}_X, A} c_{Aa} \lambda_{(a)} \partial_m B_{(A)}^m \\ &= \frac{1}{2} \sum_{a \in \mathcal{G}_X, b \in \mathcal{G}} \hat{c}_{ab} \lambda_{(a)} (F \cdot \tilde{F})_b, \quad \hat{c}_{ab} \equiv \sum_A c_{Aa} \tilde{c}_{Ab}. \end{aligned} \quad (\text{C.4})$$

The anomaly cancellation coefficients  $\hat{c}_{ab}$  can then be matched to those obtained from the underlying theory; e.g., any of the matrices enumerated in [25].

The terms (C.1) are obtained in the case where the GS counterterm Lagrangian for the anomalous  $U(1)$ 's is given by

$$\mathcal{L}_{GS}^X = - \int E \sum_{a \in \mathcal{G}_X, A} c_{Aa} L_A V_a. \quad (\text{C.5})$$

Here  $V_a$  are the vector superfields corresponding to the anomalous  $U(1)$ 's and  $L_A$  are linear superfields that arise in the twisted closed string sector. These linear multiplets are coupled to Chern-Simons superfields in a manner implied by (C.2), which leads to modified linearity conditions:

$$(\bar{\mathcal{D}}^2 - 8R)L_A = -(\mathcal{W}\mathcal{W})_A, \quad (\mathcal{D}^2 - 8\bar{R})L_A = -(\overline{\mathcal{W}\mathcal{W}})_A, \quad (\text{C.6})$$

$$[\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}]L_A = 4L_A G_{\alpha\dot{\alpha}} + 2B_{A\alpha\dot{\alpha}} + 2(\mathcal{W}_\alpha \overline{\mathcal{W}}_{\dot{\alpha}})_A, \quad (\text{C.7})$$

$$(\mathcal{W}\mathcal{W})_A = \sum_a \tilde{c}_{Aa} (\mathcal{W}\mathcal{W})_a, \quad \text{etc.} \quad (\text{C.8})$$

Corresponding to (C.5) it is necessary to generalize (1.10):

$$B_b = \sum_I (b - b_b^I) g^I - \sum_{a \in \mathcal{G}_X} \hat{c}_{ab} V_a + f_b(L). \quad (\text{C.9})$$

It is easy to see that the anomalous shift generated by (C.9) is canceled by (C.5). For suppose  $V_a \rightarrow V'_a + \frac{1}{2}(\Theta_a + \bar{\Theta}_a)$  with  $U(1)_a \in \mathcal{G}_X$ . We then have

$$\delta \mathcal{L}_Q = \int \frac{E}{16R} \sum_{a \in \mathcal{G}_X, b \in \mathcal{G}} \hat{c}_{ab} \Theta_a (\mathcal{W}\mathcal{W})_b + \text{h.c.} \quad (\text{C.10})$$

On the other hand (C.5) shifts according to

$$\begin{aligned} \delta \mathcal{L}_{GS}^X &= -\frac{1}{2} \int E \sum_{a \in \mathcal{G}_X, A} c_{Aa} L_A \Theta_a + \text{h.c.} \\ &= \int \frac{E}{16R} \sum_{a \in \mathcal{G}_X, A} c_{Aa} \Theta_a (\bar{\mathcal{D}}^2 - 8R) L_A + \text{h.c.} \\ &= -\int \frac{E}{16R} \sum_{a \in \mathcal{G}_X, b \in \mathcal{G}, A} c_{Aa} \tilde{c}_{Ab} \Theta_a (\mathcal{W}\mathcal{W})_b + \text{h.c.} \\ &= -\int \frac{E}{16R} \sum_{a \in \mathcal{G}_X, b \in \mathcal{G}} \hat{c}_{ab} \Theta_a (\mathcal{W}\mathcal{W})_b + \text{h.c.} \end{aligned} \quad (\text{C.11})$$

so that  $\delta \mathcal{L}_Q + \delta \mathcal{L}_{GS}^X = 0$ .

In other respects the effective Lagrangian can be formulated according to the approach described in [26]. From the universal dilaton linear multiplet  $L$ , other untwisted closed string linear multiplets  $L_i$  and the twisted closed string linear multiplets  $L_A$  we can form a linear combination corresponding to each factor  $\mathcal{G}_a$  of the gauge group:

$$L_a = \zeta_a^L L + \sum_i \zeta_a^i L_i + \sum_A \zeta_a^A L_A. \quad (\text{C.12})$$

The coefficients are chosen such that the  $L_a$  satisfy modified linearity conditions corresponding to a coupling to the Chern-Simons superfields of the factor  $\mathcal{G}_a$ :

$$(\bar{\mathcal{D}}^2 - 8R)L_a = -(\mathcal{W}\mathcal{W})_a, \quad (\mathcal{D}^2 - 8\bar{R})L_a = -(\overline{\mathcal{W}\mathcal{W}})_a, \quad (\text{C.13})$$

$$[\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}]L_a = 4L_a G_{\alpha\dot{\alpha}} + 2B_{(a)\alpha\dot{\alpha}} + 2(\mathcal{W}_\alpha \overline{\mathcal{W}}_{\dot{\alpha}})_a. \quad (\text{C.14})$$

Following [26], the  $L$  dependent functionals appearing in the main text can then be generalized:

$$k(L) \rightarrow \sum_a k_a(L_a), \quad 2Ls(L) \rightarrow \sum_a 2L_a s_a(L_a). \quad (\text{C.15})$$

Further details may be found in [26].

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